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Decoherence in QED at finite temperature

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Abstract. We consider a wavepacket of a charged particle passing through a cavity filled with photons at temperature T and investigate its localization and interference properties. It is shown that the wavepacket becomes localized and the interference disappears with an exponential speed after a sufficiently long path through the cavity.

1. Introduction

The photons in quantum mechanics can be considered either in an experiment performed simultaneously on photons and electrons or they can be treated as a background kept fixed during an experiment. Their role can be significant. It has been known for a long time that thermal photons can substantially disturb relativistic electron beams in an accelerator [1–3]. It also has been suggested that the problem of the classical limit of quantum mechanics (at least the problem of decoherence) can be solved in the presence of a reservoir of photons. Some models of an interaction of a quantum system with a reservoir confirming the decoherent behaviour have been discussed in [4–6]. These models are treated as an approximation (the dipole approximation) to the quantum electrodynamics (QED). Calculations in the standard QED involve a perturbation series in the fine structure constant. Eventual classical effects of the photon–electron interaction have to be estimated by an extrapolation (or resummation) of the perturbation expansion. The vacuum persistence amplitude in the relativistic QED has been studied by Ford [7]. He discussed a change of the vacuum fluctuations with a change of the environment's geometry (the Casimir effect) and its role in the interference experiments. In such a case the relative strength of the vacuum fluctuations rather than its intrinsic value (which is infinite) is relevant. A systematic approach to the particle dynamics in a relativistic QED has been developed by Anastopoulos and Zoupas [8]. These authors first derive the propagation kernel for the field theoretic density matrix and subsequently reduce it to the one particle sector.

We discussed QED at finite temperature (reduced to a fixed finite number of non-relativistic particles) and some approximations to this model in [9]. We pointed out a difficulty (resulting from the ultraviolet singularity) with some approximations to QED when applied in a discussion of the decoherence. The vacuum fluctuations lead to divergencies at short distances which require either an ultraviolet cutoff or a renormalization. It has been pointed out in [8] that the singular part of the QED finite temperature propagator (describing vacuum fluctuations) at large time does not contribute to the decoherence in the approximation of a fixed finite number of electrons.

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In this paper we concentrate on the regular part coming from the thermal bath. We calculate the time evolution of the density matrix of wavepackets in an approximation of a small charge e treating the particle dynamics in a semiclassical approximation. We neglect the vacuum fluctuations considering the effect of thermal photons on the evolution of the density matrix. The decoherence in an environment of thermal photons is demonstrated. The decoherence rate is increasing fast with an increase of time and temperature. We discuss a contribution of the vacuum fluctuations to the evolution of the density matrix (in our approximation to QED) in the appendix. It is shown that the regularized expression as well as the renormalized one vary slowly in time and space.

The effect of photons on decoherence has been discussed first by Joos and Zeh [10]. The general arrangement for interference experiments in an environment has been considered in [11]. The decoherent effect of the black body radiation has been studied by Stapp [12] (we compare our results with [12] at the end of section 3).

We are interested in QED at finite temperature T determined by the density matrix (the Gibbs state)

$$\rho = (\mathrm{Tr}(\exp(-\beta H_R)))^{-1}\exp(-\beta H_R)$$

where $\frac{1}{\beta} = KT$, K is the Boltzmann constant and H_R is the Hamiltonian for the quantum free electromagnetic field.

We compute the correlation functions explicitly:

$$G_\beta(0,\boldsymbol{x};t,\boldsymbol{x}')_{jl} \equiv \mathrm{Tr}(A_j(0,\boldsymbol{x})A_l(t,\boldsymbol{x}')\rho) = \frac{\hbar c}{\pi^2}\int \mathrm{d}\boldsymbol{k}|\boldsymbol{k}|^{-1}\cos((\boldsymbol{x}-\boldsymbol{x}')\boldsymbol{k})\delta^{\mathrm{tr}}_{jl}(\boldsymbol{k}) \times(\cos(c|\boldsymbol{k}|t)\left(\frac{1}{2}+(\exp(\beta\hbar c|\boldsymbol{k}|)-1)^{-1}\right)-\frac{\mathrm{i}}{2}\sin(c|\boldsymbol{k}|t)) \quad (1)$$

where

$$\delta^{\mathrm{tr}}_{jl}(\boldsymbol{k}) = \delta_{jl} - k_j k_l|\boldsymbol{k}|^{-2}.$$

The term $\frac{1}{2}\hbar c|\boldsymbol{k}|$ corresponds to the zero-point energy (of vacuum fluctuations) whereas $\hbar c|\boldsymbol{k}|(\exp(\hbar c|\boldsymbol{k}|\beta)-1)^{-1}$ is the average energy of thermal photons with the wavenumber $\boldsymbol{k}$. The vacuum fluctuation (noise) is a measurable effect and in general cannot be neglected. The virtual photons corresponding to the vacuum fluctuations are not directly observable. Then, the thermal photons are described by the Green function $G_{\mathrm{th}} = G_\beta - G_\infty$ (note that G_{th} is real, whereas G_β and G_∞ are complex; in quantum field theory the imaginary part of the Green function is related to the pair creation and annihilation, so subtracting G_∞ means that the processes of pair creation and annihilation are neglected)

$$G_{\mathrm{th}}(\boldsymbol{x},\boldsymbol{x}',t)_{jl} = \frac{\hbar c}{2\pi^2}\int \mathrm{d}\boldsymbol{k}|\boldsymbol{k}|^{-1}\delta^{\mathrm{tr}}_{jl}(\boldsymbol{k})\cos((\boldsymbol{x}-\boldsymbol{x}')\boldsymbol{k})\cos(ct|\boldsymbol{k}|)(\exp(\beta\hbar c|\boldsymbol{k}|)-1)^{-1}. \quad (2)$$

Let us note that G_{th} determines the real Gaussian random field. Subsequent computations can be performed either in the Fock space or by means of the functional integration. We do not explain the equivalence of both methods here but refer to our earlier paper [9].

For a small $|\boldsymbol{x}-\boldsymbol{x}'|$

$$G_{\mathrm{th}}(\boldsymbol{x},\boldsymbol{x}',t)_{jl} \simeq G_{\mathrm{th}}(0,0,t)_{jl} = \delta_{jl}\tfrac{4}{3}\hbar c\pi^{-1}\int_0^\infty \mathrm{d}k\, k\cos(ckt)(\exp(\beta\hbar ck)-1)^{-1} \quad (3)$$

is approximately $\boldsymbol{x}$-independent.

2. The semiclassical approximation

We approach the semiclassical limit of the wavefunction in the standard way treating the electromagnetic field $\boldsymbol{A}$ as a classical field. Then, the quantum electromagnetic field is realized as a random field with the covariance (2). A solution of the Schrödinger equation

$$\mathrm{i}\hbar\partial_t\psi(t,\boldsymbol{x}) = \frac{1}{2m}\left(-\mathrm{i}\hbar\nabla + \frac{e}{c}\boldsymbol{A}_t\right)^2\psi(t,\boldsymbol{x}) \tag{4}$$

with the initial condition $\psi = \exp(\frac{\mathrm{i}}{\hbar}W)\phi$ can be related to the solution W_τ of the Hamilton–Jacobi equation

$$\partial_t W_t + \frac{1}{2m}\left(\nabla W_t + \frac{e}{c}\boldsymbol{A}_t\right)^2 = 0 \tag{5}$$

with the initial condition $W_{t=0}(\boldsymbol{x}) = W(\boldsymbol{x})$. We express ψ_t in the form

$$\psi_t \equiv \chi_t\phi_t = \exp\left(\frac{\mathrm{i}}{\hbar}W_t\right)\phi_t.$$

Then, ψ_t is the solution of equation (4) if and only if ϕ_t is the solution of the equation

$$\partial_t\phi_t = \frac{\mathrm{i}\hbar}{2m}\triangle\phi_t - \frac{1}{m}\left(\nabla W_t + \frac{e}{c}\boldsymbol{A}_t\right)\nabla\phi_t - \frac{1}{2m}\left(\triangle W_t + \frac{e}{c}\mathrm{div}\boldsymbol{A}_t\right)\phi_t \tag{6}$$

with the initial condition ϕ. In a formal limit $\hbar \to 0$ the term $\triangle\phi$ can be neglected. In such an approximation the solution of the Schrödinger equation (4) is expressed by the classical flow starting from $\boldsymbol{x}$ (here $0 \leqslant s \leqslant t$)

$$\frac{\mathrm{d}\boldsymbol{y}_s}{\mathrm{d}s} = -\frac{1}{m}\left(\nabla W_{t-s}(\boldsymbol{y}_s) + \frac{e}{c}\boldsymbol{A}_{t-s}(\boldsymbol{y}_s)\right). \tag{7}$$

Until $\mathrm{o}(\hbar)$ terms we have

$$\psi(t,\boldsymbol{x}) = \exp\left(\frac{\mathrm{i}}{\hbar}W_t(\boldsymbol{x})\right)\exp\left(-\int_0^t \frac{1}{2m}\left(\triangle W_{t-s}(\boldsymbol{y}_s) + \frac{e}{c}\mathrm{div}\boldsymbol{A}_{t-s}(\boldsymbol{y}_s)\right)\mathrm{d}s\right)\phi(\boldsymbol{y}_t(\boldsymbol{x})).$$

If we know the trajectory (e.g., from the Hamilton equations) then we can compute W_t

$$W_t(\boldsymbol{x}) = W(\boldsymbol{y}_t(\boldsymbol{x})) + \int_0^t \left(\frac{m}{2}\left(\frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}s}\right)^2 + \frac{e}{c}\boldsymbol{A}_s(\boldsymbol{y}_s)\frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}s}\right)\mathrm{d}s. \tag{8}$$

From the correlation functions (2) of $\boldsymbol{A}$ it follows that the assumption that $\boldsymbol{A}(t,\boldsymbol{x})$ is $\boldsymbol{x}$-independent is a good approximation for a non-relativistic motion. For a wavepacket of momentum $\boldsymbol{p}$ we have $W = \boldsymbol{p}\boldsymbol{x}$. Hence, approximately

$$\frac{\mathrm{d}\boldsymbol{y}_s}{\mathrm{d}s} = -\frac{1}{m}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}_{t-s}\right). \tag{9}$$

This is a consistent approximation because for the approximate solution of the Hamilton–Jacobi equation $\nabla W_s(\boldsymbol{x})$ is space independent. Then, for $\boldsymbol{A}$ which is space independent we have the exact solution of equation (5)

$$W_t(\boldsymbol{x}) = \boldsymbol{p}\boldsymbol{x} - \frac{t}{2m}\boldsymbol{p}^2 - \frac{e}{2mc}\boldsymbol{p}\int_0^t \boldsymbol{A}_\tau\,\mathrm{d}\tau - \frac{e^2}{2mc^2}\int_0^t \boldsymbol{A}_\tau^2\,\mathrm{d}\tau. \tag{10}$$

As a result of the evolution in an environment of photons (which are not under an observation) the pure state

$$\psi = \exp(\mathrm{i}W/\hbar)\phi$$

after an average over the states of the quantum electromagnetic field is transformed into a mixed state with the density matrix

$$\rho_t(\boldsymbol{x}, \boldsymbol{x}') = \langle \overline{\psi_t}(\boldsymbol{x})\psi_t(\boldsymbol{x}')\rangle \tag{11}$$

where the average is over the electromagnetic field.

We combine the approximation (9), (10) of a space-independent $\boldsymbol{A}$ with the exact W_t (8) in the following approximate expression for W_t:

$$W_t(\boldsymbol{x}) = \boldsymbol{p}\boldsymbol{x} - \frac{t}{2m}\boldsymbol{p}^2 - \frac{e}{2mc}\boldsymbol{p}\int_0^t \boldsymbol{A}_\tau\left(\boldsymbol{x} - \frac{\tau}{m}\boldsymbol{p}\right)\mathrm{d}\tau. \tag{12}$$

This expression results from a solution of the equations of motion (7) to the lowest (zeroth) order in e. Subsequently, W_t in equation (8) is calculated to the first order in e. Inserting W_t (12) into equation (11) we obtain

$$\begin{aligned}\rho_t(\boldsymbol{x}, \boldsymbol{x}') &\equiv \exp((\mathrm{i}\boldsymbol{p}\boldsymbol{x}' - \mathrm{i}\boldsymbol{p}\boldsymbol{x})/\hbar)\overline{\phi\left(\boldsymbol{x} - \frac{t}{m}\boldsymbol{p}\right)}\phi\left(\boldsymbol{x}' - \frac{t}{m}\boldsymbol{p}\right)\exp(-S)\\
&= \exp((\mathrm{i}\boldsymbol{p}\boldsymbol{x}' - \mathrm{i}\boldsymbol{p}\boldsymbol{x})/\hbar)\overline{\phi\left(\boldsymbol{x} - \frac{t}{m}\boldsymbol{p}\right)}\phi\left(\boldsymbol{x}' - \frac{t}{m}\boldsymbol{p}\right)\\
&\quad\times\exp\bigg(-\frac{e^2}{m^2c^2\hbar^2}\int_0^t \boldsymbol{p}G_{\mathrm{th}}((s-\tau)\boldsymbol{p}/m, s-\tau)\boldsymbol{p}\,\mathrm{d}s\,\mathrm{d}\tau\\
&\quad+\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}G_{\mathrm{th}}(\boldsymbol{x} - \boldsymbol{x}' + (s-\tau)\boldsymbol{p}/m, s-\tau)\boldsymbol{p}\,\mathrm{d}s\,\mathrm{d}\tau\\
&\quad+\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}G_{\mathrm{th}}(\boldsymbol{x}' - \boldsymbol{x} + (s-\tau)\boldsymbol{p}/m, s-\tau)\boldsymbol{p}\,\mathrm{d}s\,\mathrm{d}\tau\bigg)\end{aligned} \tag{13}$$

where $\exp(-S)$ denotes the last factor in equation (13). If the vacuum fluctuations were to be taken into account then we would need to make the replacement $G_{\mathrm{th}} \to G_\beta = G_{\mathrm{th}} + \mathrm{i}\triangle_{\mathrm{F}}$, where $\triangle_{\mathrm{F}}$ is the Feynman causal propagator (in the notation of Bjorken and Drell [13]). However, this term gives a negligible contribution to the decoherence for a large time. A detailed discussion of the possible effect on decoherence of vacuum fluctuations, electron–positon pair annihilation and photon emission in zero-temperature QED can be found in [8]. We discuss the contribution of vacuum fluctuations to equation (13) in the appendix. We show that this contribution is negligible because it is slowly varying in space and time $t > 0$. Hence, at higher temperature and longer time the thermal part is dominating.

3. The estimates of the evolution of the density matrix

We perform first the integral over time in equation (13). We denote $\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{x}'$, introduce the spherical coordinates $\mathrm{d}\boldsymbol{k} = \mathrm{d}k\,k^2\,\mathrm{d}\theta\sin\theta\,\mathrm{d}\phi$ and write S in the form

$$S = \frac{e^2}{2\pi m^2 c\hbar}\int_0^\infty \mathrm{d}k\,k\int_0^\pi \mathrm{d}\theta\sin\theta\, I(\theta, k, p)(\exp(\beta\hbar ck) - 1)^{-1}. \tag{14}$$

We restrict ourselves to $|\boldsymbol{p}| \ll mc$ then

$$\begin{aligned}I(\theta, k, p) &= c^{-2}k^{-2}|\boldsymbol{p}|^2(1-\cos^2\theta)(2(1-\cos(tck))\\
&\quad -2\cos\boldsymbol{k}\boldsymbol{y} + \cos(\boldsymbol{k}\boldsymbol{y} + ckt) + \cos(\boldsymbol{k}\boldsymbol{y} - ckt)).\end{aligned} \tag{15}$$

The integral (14) depends on a parameter with the dimension of a length

$$l_{\mathrm{dB}} = c\hbar\beta$$

which is called the thermal de Broglie wavelength at temperature T. We could make the rescaling $k \to k l_{dB}^{-1}$ then the length would be measured in terms of l_{dB}. We shall write that $|\boldsymbol{y}|$ is large if $|\boldsymbol{y}| l_{dB}^{-1} \gg 1$, similarly time t is large if $ct l_{dB}^{-1} \gg 1$.

If $\boldsymbol{y}$ is large then on the basis of equation (15) the $\boldsymbol{y}$-dependent terms are small as a function of t in comparison with other terms, because we have an additional oscillation in $\boldsymbol{y}$ which makes such a term negligible (unless $|\boldsymbol{y}|$ is comparable with ct). The main contribution to S for a large $\boldsymbol{y} \gg ct$ is (we write $\alpha = \cos\theta$)

$$S = \frac{2e^2}{m^2 c^3 \hbar \pi} \boldsymbol{p}^2 \int_{-1}^{1} \mathrm{d}\alpha (1-\alpha^2) \int_0^\infty \mathrm{d}k\, k^{-1} (1 - \cos(tck)) (\exp(\beta \hbar c k) - 1)^{-1}. \tag{16}$$

For a small time t (and a large $\boldsymbol{y}$) we obtain

$$\begin{aligned} |\rho_t| &\approx \exp\left(-\frac{2}{3} \frac{e^2}{m^2 \hbar c \pi} \boldsymbol{p}^2 t^2 \int_0^\infty \mathrm{d}k\, k (\exp(\beta \hbar c k) - 1)^{-1} \right) \\ &\approx \exp\left(-B \frac{e^2}{\hbar c} (\boldsymbol{p} t/m)^2 l_{dB}^{-2} \right) \end{aligned} \tag{17}$$

where B is a constant of order 1. The result (17) means that the decoherence (or simply the effect of the electromagnetic environment) is visible after a particle makes a path comparable with the de Broglie wavelength.

Let us consider now a large $t \geqslant 0$ in equation (16). We apply the formula

$$1 - \cos w = w \int_0^1 \mathrm{d}\gamma \sin(\gamma w)$$

and the formula (3.911) of Gradshtein and Ryzhik [15]

$$\int_0^\infty \mathrm{d}u \sin(au) (\exp(\beta u) - 1)^{-1} = \frac{\pi}{2\beta} \coth\left(\frac{\pi a}{\beta}\right) - \frac{1}{2a}.$$

Then, we obtain

$$\begin{aligned} S &= \frac{4}{3} \frac{t e^2}{m^2 c^2 \hbar \pi} \boldsymbol{p}^2 \int_0^1 \mathrm{d}\gamma \int_0^\infty \mathrm{d}k \sin(tck\gamma) (\exp(\beta \hbar c k) - 1)^{-1} \\ &= \frac{4}{3} \frac{e^2}{m^2 c^3 \hbar \pi} \boldsymbol{p}^2 \int_0^t \mathrm{d}\gamma \left(\frac{\pi}{\beta \hbar} \coth\left(\frac{\pi \gamma}{\beta \hbar} \right) - \frac{1}{\gamma} \right) \\ &= \frac{4}{3} \frac{e^2}{m^2 c^3 \hbar \pi} \boldsymbol{p}^2 \ln\left(\frac{\beta \hbar}{\pi t} \sinh\left(\frac{\pi t}{\beta \hbar} \right) \right) \approx \frac{|\boldsymbol{p}|^2}{m^2 c^2} \frac{e^2}{c \hbar} \frac{ct}{l_{dB}} \end{aligned} \tag{18}$$

for a large t such that $|\boldsymbol{y}| \gg ct \gg l_{dB}$.

Let us consider next small t together with a small $\boldsymbol{y}$. Then, we can set $s = \tau = 0$ in the argument of G_{th} in equation (13). In such a case

$$S \simeq \frac{e^2}{m^2 c \hbar \pi} \boldsymbol{p}^2 t^2 \int_0^\infty \mathrm{d}k\, k \int_0^\pi \mathrm{d}\theta \sin\theta (1 - \cos^2\theta)(1 - \cos(k \cos\theta |\boldsymbol{y}|))(\exp(\beta \hbar c k) - 1)^{-1}. \tag{19}$$

Formula (19) is relevant only for a small $\boldsymbol{y}$ (for a large $\boldsymbol{y}$ we return to equation (17)). From equation (19) for a small $\boldsymbol{y}$ and small t we obtain

$$\begin{aligned} \rho_t(\boldsymbol{x}, \boldsymbol{x}') &\approx \exp\left(-\frac{2e^2}{15 \pi m^2 \hbar c} \boldsymbol{p}^2 t^2 \boldsymbol{y}^2 \int_0^\infty \mathrm{d}k\, k^3 (\exp(\hbar c \beta k) - 1)^{-1} \right) \\ &\approx \exp\left(-B \frac{e^2}{\hbar c} \left(\frac{\boldsymbol{p} t}{m} \right)^2 \boldsymbol{y}^2 l_{dB}^{-4} \right) \end{aligned} \tag{20}$$

where B is a constant of order 1. Again we can conclude that the decoherence is visible on distances comparable to the thermal de Broglie length.

For a large $t \gg |\boldsymbol{y}|/c$ we neglect the quickly oscillating t-dependent terms in equation (15). Then, similarly as in the computations in equation (18) (with $\alpha = \cos\theta$)

$$\begin{aligned} S &= \frac{e^2}{\pi m^2 c^3 \hbar} \boldsymbol{p}^2 |\boldsymbol{y}| \int_0^1 \mathrm{d}\gamma \int_{-1}^1 \mathrm{d}\alpha\alpha(1-\alpha^2) \int_0^\infty \mathrm{d}k (\exp(\beta\hbar c k) - 1)^{-1} \sin(k\gamma\alpha|\boldsymbol{y}|) \\ &= \frac{2e^2}{\pi m^2 c^3 \hbar} \boldsymbol{p}^2 \int_0^1 \mathrm{d}\alpha(1-\alpha^2) \ln\left(\frac{\beta\hbar c}{\pi\alpha|\boldsymbol{y}|} \sinh\left(\frac{\pi\alpha|\boldsymbol{y}|}{\beta\hbar c}\right)\right) \approx \frac{e^2}{\hbar c} \frac{\boldsymbol{p}^2}{m^2 c^2} \frac{|\boldsymbol{y}|}{l_{\mathrm{dB}}} \end{aligned} \quad (21)$$

for a large $\boldsymbol{y}$ such that $ct \gg |\boldsymbol{y}| \gg l_{\mathrm{dB}}$.

For a small $|\boldsymbol{y}|$ (and a large t) the t-dependent terms in equation (15) can be neglected. Then, the integral (14) gives

$$S = \frac{2}{15} \frac{e^2}{\pi m^2 c^3 \hbar} |\boldsymbol{p}|^2 |\boldsymbol{y}|^2 l_{\mathrm{dB}}^{-2} \int_0^\infty \mathrm{d}k\, k(\exp k - 1)^{-1}.$$

Hence,

$$|\rho_t| \approx \exp\left(-B\frac{e^2}{\hbar c} |\boldsymbol{p}|^2 (mc)^{-2} |\boldsymbol{y}|^2 l_{\mathrm{dB}}^{-2}\right).$$

At this point it is useful to recall the definition of the Wigner function

$$\mathcal{W}(\boldsymbol{q}, \boldsymbol{k}) = (2\pi\hbar)^{-3} \int \mathrm{d}\boldsymbol{u} \exp(\mathrm{i}\boldsymbol{k}\boldsymbol{u}/\hbar) \rho(\boldsymbol{q} + \boldsymbol{u}/2, \boldsymbol{q} - \boldsymbol{u}/2).$$

If $\rho \approx \exp(-\frac{a}{2}\boldsymbol{y}^2 - \mathrm{i}py/\hbar)$ then

$$\mathcal{W}(\boldsymbol{q}, \boldsymbol{k}) \simeq \exp\left(-\frac{1}{2a\hbar^2} (\boldsymbol{p} - \boldsymbol{k})^2\right) w(\boldsymbol{q}, \boldsymbol{k}).$$

Such a behaviour means a localization on the classical momentum $\boldsymbol{p}$. If (as in equation (18) for a large time) $\rho_t = \exp(-\mathrm{i}\boldsymbol{p}\boldsymbol{y} - S) \simeq \exp(-\mathrm{i}\boldsymbol{p}\boldsymbol{y} - b\boldsymbol{p}^2 t)$ then approximately

$$\partial_t \rho_t \approx -b[\boldsymbol{P}, [\boldsymbol{P}, \rho_t]] \quad (22)$$

where $\boldsymbol{P} = -\mathrm{i}\hbar\nabla$ is the quantum momentum operator. Then

$$\partial_t \mathcal{W}_t(\boldsymbol{q}, \boldsymbol{k}) \approx -b\boldsymbol{k}^2 \mathcal{W}_t(\boldsymbol{q}, \boldsymbol{k}).$$

Let us note that the behaviour (18) of ρ_t could have been obtained from equation (13) if we made the approximation

$$G_{\mathrm{th}}(\boldsymbol{x} - \boldsymbol{x}', s - \tau) \approx \hbar l_{\mathrm{dB}}^{-1} \delta(\tau - s).$$

Then, the Schrödinger equation (4) would be approximated by the stochastic Schrödinger equation [16, 17]

$$\partial_t \psi = \frac{\mathrm{i}\hbar}{2m} \triangle\psi - \frac{\mathrm{i}e}{mc\hbar} \boldsymbol{A}\boldsymbol{P}\psi$$

where $\boldsymbol{A}$ is approximated by the white noise (as a consequence we obtain equation (22)). Such an approximation for the relict radiation has been discussed by Stapp [12]. It is difficult to justify it starting from the expressions (2), (3) for G_{th}. From our results it follows that it can approximate the behaviour of $\rho_t(\boldsymbol{x}, \boldsymbol{x}')$ only if $|\boldsymbol{x} - \boldsymbol{x}'| \gg ct \gg l_{\mathrm{dB}}$.

For N-particles with momenta $\boldsymbol{p}_j$ we consider the initial state of the form

$$\psi = \exp\left(\frac{\mathrm{i}}{\hbar} \sum_{j=1}^N \boldsymbol{p}_j \boldsymbol{x}_j\right) \phi.$$

Then, W_t in equation (12) is a sum of the Hamilton–Jacobi functions for each particle. The subsequent expectation value over the electromagnetic field gives an exponential with a sum of pairings between the N particles. Hence, it can be expected that ρ_t behaves as $\exp(-RN^2t^2)$ for a small t and as $\exp(-RN^2|t|)$ for a large t with a certain constant R (note that the behaviour $\exp(-RN|t|)$ for another model has been obtained by Unruh [14]). Such a result is valid for N particles of equal charges under the assumption that the terms in the exponential of the form (13) have equal signs. The contributions add if the momenta have a distinguished direction. If the directions are random then the contributions from different particles cancel one another. A similar cancellation takes place if the system is neutral, i.e. if the charges e_j can have different signs. Then, in the sum in an exponential of the form (13) we shall have $e_j e_k \boldsymbol{p}_j \boldsymbol{p}_k$ multiplying G_{th}. Hence, there can be cancellations from different charges e_j even if there is a distinguished direction of the momenta. Under an assumption that the contributions in the exponential do not cancel one another we obtain for a large distance between any pair of coordinates and a large t

$$|\rho_t| \approx \exp\left(-B\frac{e^2}{c\hbar}N^2\left(\frac{|\boldsymbol{p}|}{mc}\right)^2\frac{ct}{l_{\text{dB}}}\right)$$

where B is a constant of order 1. The decay is visible at time t if

$$N^2 t > \frac{c\hbar}{e^2}\left(\frac{mc}{|\boldsymbol{p}|}\right)^2\frac{l_{\text{dB}}}{c}.$$

This time can be short if N is large.

4. Disappearance of the interference

So far we have discussed the decay of $\rho_t(\boldsymbol{x}, \boldsymbol{x}')$ for varying t and $\boldsymbol{y} = \boldsymbol{x}-\boldsymbol{x}'$. The disappearance of the off-diagonal matrix elements indicates a classical behaviour of quantum probabilities in the state ρ_t. We investigate next what happens with the interference describing a typical quantum phenomenon. We consider a superposition of two wavepackets

$$\psi(\boldsymbol{x}) = \exp(\mathrm{i}\boldsymbol{p}^{(1)}\boldsymbol{x}/\hbar)\phi^{(1)}(\boldsymbol{x}) + \exp(\mathrm{i}\boldsymbol{p}^{(2)}\boldsymbol{x}/\hbar)\phi^{(2)}(\boldsymbol{x}).$$

Then, at the point $\boldsymbol{x}$ (on the screen) after an evolution through a cavity filled with thermal photons the probability density $\langle|\psi_t(\boldsymbol{x})|^2\rangle$ is equal to the diagonal part of the density matrix

$$\rho_t = \langle|\psi_t\rangle\langle\psi_t|\rangle. \tag{23}$$

For each packet we have the Hamilton–Jacobi function

$$W_t(\boldsymbol{x}) = \boldsymbol{p}\boldsymbol{x} - t\frac{\boldsymbol{p}^2}{2m} - \frac{\boldsymbol{p}e}{mc}\int_0^t \boldsymbol{A}(\boldsymbol{y}_s(\boldsymbol{x}), s)\,\mathrm{d}s. \tag{24}$$

Under the time evolution

$$\psi \to \psi_t = \exp(\mathrm{i}W_t^{(1)}/\hbar)\phi_t^{(1)} + \exp(\mathrm{i}W_t^{(2)}/\hbar)\phi_t^{(2)}.$$

In our semiclassical approximation $\phi(\boldsymbol{x}) \to \phi(\boldsymbol{y}_t(\boldsymbol{x}))$. Then, for weak fields we neglect the dependence of the paths on the electromagnetic field, i.e. we consider straight lines

$$\boldsymbol{y}_s^{(1)} = \boldsymbol{x} - \frac{s}{m}\boldsymbol{p}^{(1)}$$

and

$$\boldsymbol{y}_s^{(2)} = \boldsymbol{x} - \frac{s}{m}\boldsymbol{p}^{(2)}.$$

In this approximation the expectation value (23) is

$$\begin{aligned}
\langle|\psi_t(\boldsymbol{x})|^2\rangle = &\left|\phi^{(1)}\left(\boldsymbol{x}-\frac{t}{m}\boldsymbol{p}^{(1)}\right)\right|^2 + \left|\phi^{(2)}\left(\boldsymbol{x}-\frac{t}{m}\boldsymbol{p}^{(2)}\right)\right|^2 \\
&+\left(\overline{\phi^{(2)}\left(\boldsymbol{x}-\frac{t}{m}\boldsymbol{p}^{(2)}\right)}\phi^{(1)}\left(\boldsymbol{x}-\frac{t}{m}\boldsymbol{p}^{(1)}\right)\right. \\
&\times\exp\left(-\frac{\mathrm{i}}{\hbar}(\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)})\boldsymbol{x}+\frac{\mathrm{i}t}{2m\hbar}((\boldsymbol{p}^{(2)})^2-(\boldsymbol{p}^{(1)})^2)\right) \\
&+\exp\left(\frac{\mathrm{i}}{\hbar}(\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)})\boldsymbol{x}-\frac{\mathrm{i}t}{2m\hbar}((\boldsymbol{p}^{(2)})^2-(\boldsymbol{p}^{(1)})^2)\right) \\
&\left.\times\overline{\phi^{(1)}\left(\boldsymbol{x}-\frac{t}{m}\boldsymbol{p}^{(1)}\right)}\phi^{(2)}\left(\boldsymbol{x}-\frac{t}{m}\boldsymbol{p}^{(2)}\right)\right) \\
&\times\exp\left(\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}^{(1)}G_{\mathrm{th}}\left(\frac{s}{m}\boldsymbol{p}^{(1)}-\frac{\tau}{m}\boldsymbol{p}^{(2)},s-\tau\right)\boldsymbol{p}^{(2)}\,\mathrm{d}s\,\mathrm{d}\tau\right. \\
&+\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}^{(1)}G_{\mathrm{th}}(\frac{s}{m}\boldsymbol{p}^{(2)}-\frac{\tau}{m}\boldsymbol{p}^{(1)},s-\tau)\boldsymbol{p}^{(2)}\,\mathrm{d}s\,\mathrm{d}\tau \\
&-\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}^{(1)}G_{\mathrm{th}}\left(\frac{s}{m}\boldsymbol{p}^{(1)}-\frac{\tau}{m}\boldsymbol{p}^{(1)},s-\tau\right)\boldsymbol{p}^{(1)}\,\mathrm{d}s\,\mathrm{d}\tau \\
&\left.-\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}^{(2)}G_{\mathrm{th}}\left(\frac{s}{m}\boldsymbol{p}^{(2)}-\frac{\tau}{m}\boldsymbol{p}^{(2)},s-\tau\right)\boldsymbol{p}^{(2)}\,\mathrm{d}s\,\mathrm{d}\tau\right) \\
\equiv\ &\rho_t^{(1)}+\rho_t^{(2)}+\rho_t^{(12)}.
\end{aligned} \tag{25}$$

Without detailed calculations we can obtain the behaviour for a small t. The timescale is again determined by the de Broglie thermal wavelength l_{dB}. However, in G_{th} in equation (25) the length enters as a particle path. Hence, time s is small if $s|\boldsymbol{p}^{(k)}|/m \ll l_{\mathrm{dB}}$ for each k. Let $s=\tau=0$ in G_{th} in equation (25), then from equation (3) $G_{\mathrm{th}}(0,0)\simeq B\delta_{jl}\beta^{-2}\hbar^{-1}c^{-1}$. It follows that

$$|\rho_t^{(12)}|\simeq\exp\left(-B\frac{e^2}{\hbar c}l_{\mathrm{dB}}^{-2}(t|\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)}|/m)^2\right). \tag{26}$$

Hence, if the decoherence is to be visible the distance between the particles after time t must be of the order of the de Broglie length. The calculations for a large time are more involved. Let us denote

$$|\rho_t^{(12)}|\equiv\exp(-S_{12}).$$

We obtain for S_{12}

$$\begin{aligned}
S_{12} &= \frac{2}{3}\frac{e^2}{\pi m^2c^3\hbar}|\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)}|^2\int_0^\infty\frac{\mathrm{d}k}{k}(\exp(\beta\hbar ck)-1)^{-1}(1-\cos(tck)) \\
&= \frac{2}{3}\frac{e^2}{\pi m^2c^3\hbar}|\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)}|^2ct\int_0^1\mathrm{d}\gamma\int_0^\infty\mathrm{d}k(\exp(\beta\hbar ck)-1)^{-1}\sin(\gamma ckt) \\
&= \frac{2}{3}\frac{e^2}{\pi m^2c^2\hbar}|\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)}|^2\int_0^1\mathrm{d}\gamma\left(\frac{t\pi}{2\beta\hbar c}\coth\left(\frac{\pi\gamma t}{\beta\hbar}\right)-\frac{1}{2\gamma c}\right) \\
&= \frac{1}{3}\frac{e^2}{\pi m^2c^3\hbar}|\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)}|^2\ln\left(\frac{\beta\hbar}{\pi t}\sinh\left(\frac{t\pi}{\beta\hbar}\right)\right) \\
&\approx \frac{e^2}{\hbar c}\frac{ct}{l_{\mathrm{dB}}}|\boldsymbol{p}^{(2)}-\boldsymbol{p}^{(1)}|^2m^{-2}c^{-2}
\end{aligned} \tag{27}$$

for a large t. Hence, the mixed term in equation (25) is multiplied by $\exp(-S_{12})$ which decays as $\exp(-Rt)$. Such a behaviour of the probability density proves that the thermal photons lead to the classical addition of probabilities instead of the quantum addition of amplitudes showing the decoherence phenomenon in a physical model of an electron–photon interaction.

5. Conclusions

We have discussed a model of a quantum charged particle interacting with a quantum electromagnetic field at finite temperature. We have calculated the time evolution of the density matrix in a semiclassical approximation for the wavefunction and in a weak coupling approximation for the particle–photon interaction. In contradistinction to [4–6] we do not apply the approximation of a linear coordinate coupling to the environment (rejected also in [8]). For a small time and small space separations we obtain an exponential in time and space decay $\rho_t \approx \exp(-bt^2|\boldsymbol{x}-\boldsymbol{x}'|^2)$ of off-diagonal matrix elements (decoherence). For a large time the decay achieves its stationary value (18) and (21). The time and space scale is determined by the thermal de Broglie wavelength. If we have a large number N of charged particles then the decoherence rate can increase as N^2. The density matrix elements decay as $\rho_t \approx \exp(-bt)$ for a large time. Such a behaviour is in agreement with the Lindblad dynamics if the dissipative part of the dynamics has the form (22). This form of the Lindblad dynamics in zero temperature QED has also been derived in [8] but with a coefficient vanishing at large time. Lindblad dynamics of such a form has been discussed earlier in [18, 19]. The Lindblad dynamics resulting from a linear coordinate coupling to the environment (studied in [4,5]) is of a different type. It could be described by a replacement of the momentum operator by a position operator in equation (22) (a coupling to the environment linear in the momentum as well as in the coordinate has been discussed by Leggett in [20]). We have studied the interference as another typical aspect of a quantum behaviour. We have shown that in an environment of photons the interference disappears with an exponential speed (equations (26), (27)). Our results suggest an arrangement for an experiment. Such experiments could verify the QED beyond the usual perturbative approximation as well as the principle of the wavefunction reduction in a non-selective measurement.

Appendix

When the operator formalism is applied then formula (13) results from a representation of the density matrix expectation values by the expectation values of the time-ordered products of quantum fields in the Fock space. We use the following conventions of Bjorken and Drell [13] (T denotes the time-ordered product of vector fields)

$$\langle 0|T(A(x')A(x))|0\rangle = \mathrm{i}\triangle_{\mathrm{F}}(x'-x)$$
$$\langle 0|T(\exp(\mathrm{i}AJ))|0\rangle = \exp\left(-\frac{\mathrm{i}}{2}J\triangle_{\mathrm{F}}J\right).$$

Then, in our notation

$$G_{\mathrm{F}}(x'-x) = \mathrm{i}\triangle_{\mathrm{F}}(x'-x).$$

In terms of Fourier integrals

$$\triangle_{\mathrm{F}}(x'-x) = -\mathrm{i}\hbar c\tfrac{1}{2}(2\pi)^{-3}\int \mathrm{d}\boldsymbol{k}|\boldsymbol{k}|^{-1}\delta^{\mathrm{tr}}(\boldsymbol{k})\cos(\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x}))\exp(-\mathrm{i}c|\boldsymbol{k}||t'-t|).$$

Then, in equation (13) $G_{\mathrm{th}} \to G_\beta = G_{\mathrm{th}} + G_{\mathrm{F}}$. Hence, $S \to S + S_{\mathrm{F}}$ where

$$
\begin{aligned}
S_{\mathrm{F}} = {}& \frac{e^2}{m^2c^2\hbar^2}\int_0^t \boldsymbol{p}G_{\mathrm{F}}((s-\tau)\boldsymbol{p}/m, s-\tau)\boldsymbol{p}\,\mathrm{d}s\,\mathrm{d}\tau \\
& -\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}G_{\mathrm{F}}(\boldsymbol{x}-\boldsymbol{x}'+(s-\tau)\boldsymbol{p}/m, s-\tau)\boldsymbol{p}\,\mathrm{d}s\,\mathrm{d}\tau \\
& -\frac{e^2}{2m^2c^2\hbar^2}\int_0^t \boldsymbol{p}G_{\mathrm{F}}(\boldsymbol{x}'-\boldsymbol{x}+(s-\tau)\boldsymbol{p}/m, s-\tau)\boldsymbol{p}\,\mathrm{d}s\,\mathrm{d}\tau \\
= {}& \frac{e^2}{m^2c^2\hbar^2}\boldsymbol{p}^2\hbar c\frac{1}{2}(2\pi)^{-3}(-\mathrm{i})ct\int \mathrm{d}\boldsymbol{k}(c^2|\boldsymbol{k}|^2-(\boldsymbol{pk})^2/m^2)^{-1} \\
& \times(1-\cos(\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x}))) \\
& +\frac{e^2}{m^2c^2\hbar^2}\boldsymbol{p}^2\hbar c\frac{1}{2}(2\pi)^{-3}\int \mathrm{d}\boldsymbol{k}|\boldsymbol{k}|^{-1}((c|\boldsymbol{k}|+\boldsymbol{pk}/m)^{-2} \\
& \times(1-\exp(-\mathrm{i}t(c|\boldsymbol{k}|+\boldsymbol{pk}/m))) \\
& +(c|\boldsymbol{k}|-\boldsymbol{pk}/m)^{-2}(1-\exp\mathrm{i}t(c|\boldsymbol{k}|-\boldsymbol{pk}/m)))(1-\cos(\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x}))).
\end{aligned}
\tag{28}
$$

The integrals are finite if we impose the ultraviolet cutoff $|\boldsymbol{k}| \leqslant \Lambda$. It supplies a length scale Λ^{-1}. With the cutoff $|\exp(-S_{\mathrm{F}})|$ behaves as follows: if $t \gg |\boldsymbol{x}-\boldsymbol{x}'|/c$ and $|\boldsymbol{x}-\boldsymbol{x}'| < \Lambda^{-1}$ then

$$|\exp(-S_{\mathrm{F}})| \approx \exp(-a|\boldsymbol{x}-\boldsymbol{x}'|^2).$$

If $|\boldsymbol{x}-\boldsymbol{x}'| \gg c|t|$ and $|t| < (c\Lambda)^{-1}$ then

$$|\exp(-S_{\mathrm{F}})| \approx \exp(-ac^2|t|^2).$$

If $|\boldsymbol{x}-\boldsymbol{x}'| < \Lambda^{-1}$ and $|t| < (c\Lambda)^{-1}$ then

$$|\exp(-S_{\mathrm{F}})| \approx \exp(-bc^2t^2|\boldsymbol{x}-\boldsymbol{x}'|^2)$$

with certain constants a and b. When $t \to \infty$ or $|\boldsymbol{x}-\boldsymbol{x}'| \to \infty$ then $\mathrm{Re}\,S_{\mathrm{F}}$ tends (logarithmically) to a constant on the basis of the Lebesgue theorem. We can choose the length scale Λ^{-1} arbitrarily small then the variation of S_{F} is negligible in comparison to the variation of the thermal part.

We can renormalize S_{F} by a subtraction of an infinite part S_c:

$$
\begin{aligned}
S_c = {}& \frac{e^2}{m^2c^2\hbar^2}\boldsymbol{p}^2\hbar c\frac{1}{2}(2\pi)^{-3}(-\mathrm{i})ct\int \mathrm{d}\boldsymbol{k}(c^2|\boldsymbol{k}|^2-(\boldsymbol{pk})^2/m^2)^{-1} \\
& +\frac{e^2}{m^2c^2\hbar^2}\boldsymbol{p}^2\hbar c\frac{1}{2}(2\pi)^{-3}\int \mathrm{d}\boldsymbol{k}|\boldsymbol{k}|^{-1}((c|\boldsymbol{k}|+\boldsymbol{pk}/m)^{-2} \\
& +(c|\boldsymbol{k}|-\boldsymbol{pk}/m)^{-2}).
\end{aligned}
\tag{29}
$$

It seems that in $\exp(-S)$ the first term in equation (29) (being purely imaginary) could be absorbed into the charge renormalization of the wavepacket in equations (12), (13) whereas the second one could be interpreted as the wavefunction renormalization (a renormalization of the normalization constant for the wavepacket; however, it remains unclear whether this renormalization coincides with the one of the complete relativistic QED). After the renormalization we obtain

$$
\begin{aligned}
S_{\mathrm{F}}^{(\mathrm{ren})} = {}& \frac{e^2}{m^2c^2\hbar^2}\boldsymbol{p}^2\hbar c\frac{1}{2}(2\pi)^{-3}\mathrm{i}ct\int \mathrm{d}\boldsymbol{k}((c^2|\boldsymbol{k}|^2-(\boldsymbol{pk})^2/m^2)^{-1}\cos(\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x}))) \\
& -\frac{e^2}{m^2c^2\hbar^2}\boldsymbol{p}^2\hbar c\frac{1}{2}(2\pi)^{-3}\int \mathrm{d}\boldsymbol{k}|\boldsymbol{k}|^{-1}((c|\boldsymbol{k}|+\boldsymbol{pk}/m)^{-2}
\end{aligned}
$$

$$\times(1-\exp(-\mathrm{i}t(c|\boldsymbol{k}|+\boldsymbol{pk}/m)))$$
$$+(c|\boldsymbol{k}|-\boldsymbol{pk}/m)^{-2}(1-\exp\mathrm{i}t(c|\boldsymbol{k}|-\boldsymbol{pk}/m)))\cos(\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x}))$$
$$-\frac{e^2}{m^2c^2\hbar^2}\boldsymbol{p}^2\hbar c\frac{1}{2}(2\pi)^{-3}\int \mathrm{d}\boldsymbol{k}|\boldsymbol{k}|^{-1}((c|\boldsymbol{k}|+\boldsymbol{pk}/m)^{-2}$$
$$\times\exp(-\mathrm{i}t(c|\boldsymbol{k}|+\boldsymbol{pk}/m))$$
$$+(c|\boldsymbol{k}|-\boldsymbol{pk}/m)^{-2}\exp\mathrm{i}t(c|\boldsymbol{k}|-\boldsymbol{pk}/m)). \tag{30}$$

This expression is finite if $t \neq 0$ and $\boldsymbol{x} \neq \boldsymbol{x}'$ owing to the oscillations of the trigonometric functions. However, we obtain an infinite expression for $t = 0$ or $\boldsymbol{x} = \boldsymbol{x}'$.

If $t \gg |\boldsymbol{x}-\boldsymbol{x}'|/c > 0$ or $|\boldsymbol{x}-\boldsymbol{x}'| \gg ct > 0$ then Re $S_{\mathrm{F}}^{(\mathrm{ren})} \approx \ln(ct|\boldsymbol{x}-\boldsymbol{x}'|^{-1})$. Such a slow logarithmic variation holds true close to the light cone as well. When $t \to 0$ or $|\boldsymbol{x}-\boldsymbol{x}'| \to 0$ then Re $S_{\mathrm{F}}^{(\mathrm{ren})}$ diverges logarithmically. We can conclude that the slow logarithmic variation of Re S_{F} for $t \neq 0$ and $\boldsymbol{x} \neq \boldsymbol{x}'$ can be neglected in comparison to the faster variation of the thermal part.

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